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NN and $N\Delta$ Form Factors viewed from ChPT

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Abstract. I discuss recent work on nucleon form factors, magnetic strangeness in the nucleon and the isovector nucleon-delta transition based on the broken chiral symmetry of QCD utilizing recent theoretical developments in ChPT.

1 Introduction

During this workshop we have heard both theoretical and experimental presentations looking for the onset of *perturbative* QCD formulated in explicit (current) quark and gluon degrees of freedom at moderate/high four-momentum transfer, *e.g.* $Q^2 > 1$ GeV^2. At lower momentum transfer one cannot avoid the complications of the strong coupling regime. In this talk I want to discuss some of the constraints resulting from the broken chiral symmetry of QCD for baryon form factors in the *non-perturbative* regime of QCD [1, 2, 3].

2 Nucleon Form Factors and ChPT

The chiral symmetry of the light flavor sector of QCD is spontaneously broken at low energies leading to the existence of Goldstone boson modes. Here we focus on the (u,d)-quark sector only which leads to the identification of the pions as the Goldstone Bosons. All low energy dynamics is governed by these lightest hadronic degrees of freedom and the chiral symmetry puts very strict constraints on their interactions among themselves, with external sources and on their coupling to matter fields (baryons, etc.). This Goldstone-boson dominated regime of *non-perturbative* QCD at low energies can be formulated exactly in an effective lagrangian formalism called Chiral Perturbation Theory (ChPT) [4]. With the pions being the lightest degrees of freedom in the hadron spectrum, ChPT suggests that the long range structure of baryons and

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its leading momentum dependence is governed by the chiral symmetry of the pion interaction.

The electro-weak structure of baryons is parameterized via form factors. In the case of the nucleon they have been analyzed in one-loop relativistic baryon ChPT [5] and in a non-relativistic approach called HBChPT [6] in the past. Recently [1], we have repeated this analysis utilizing a phenomenological extension of ChPT called the "small scale expansion" [7]. In this approach one includes the first nucleon resonance $\Delta(1232)$ as an explicit degree of freedom in a phenomenologically resummed chiral expansion. In [1] all 6 form factors of the nucleon are discussed, here I will only address the isovector Pauli form factor $F_2^v(q^2)$.

Consider the nucleon matrix element of the isovector component of the quark vector current $V_\mu^i = \bar{q}\gamma_\mu(\tau^i/2)q$, which involves a vector (Dirac) and a tensor (Pauli) form factor,

$$\langle N(p_2)|V_\mu^i(0)|N(p_1)\rangle = \bar{u}(p_2)\left[F_1^{\,v}(q^2)\,\gamma_\mu + \frac{i}{2M_N}F_2^{\,v}(q^2)\,\sigma_{\mu\nu}q^\nu\right]u(p_1)\,,\ (2.1)$$

where $u(p)$ is a Dirac spinor and $q^2 = (p_2 - p_1)^2$ is the invariant momentum transfer squared. The radii of these form factors should be determined by the extension of the pion cloud. E.g. for the radius of $F_2^v(q^2)$ one finds to leading order in HBChPT [6], [SSE [1]]

$$\begin{aligned}
(r_2^v)^2 &= \frac{g_A^2 M_N}{8F_\pi^2\kappa_v\pi m_\pi} + \left[\frac{g_{\pi N\Delta}^2 M_N}{9F_\pi^2\kappa_v\pi^2\sqrt{\Delta^2 - m_\pi^2}}\log\left[\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}\right]\right]\\
&= 0.52\text{fm}^2\left[+\,0.09\text{fm}^2\right]\,, \hspace{4cm}(2.2)
\end{aligned}$$

compared with the empirical value, $(r_2^v)^2 = 0.80\text{fm}^2$ [8]. The only parameters are the pion decay constant (mass) F_π (m_π), the πNN $(\pi\Delta N)$ couplings g_A $(g_{\pi N\Delta})$, the nucleon mass M_N, the mass-splitting $\Delta = M_\Delta - M_N$ and the anomalous isovector magnetic moment κ_v. One can see that already the *leading* HBChPT result for the extension of the pion cloud provides a good estimate for the size of the nucleon in this channel. Inclusion of explicit delta components in the nucleon wavefunction around which the pions can fluctuate provides a 17% correction in the right direction [1]. In the chiral limit, we recover the well known $1/m_\pi$ singularity, which is not touched by the resonance contribution in accord with general decoupling requirements. Other form factor results are discussed in [1].

3   Magnetic Strangeness in the Nucleon

So far we have focused on baryon ChPT involving two light flavors. The analysis presented in the previous section can be generalized in a straightforward fashion to a SU(3) chiral symmetry of QCD, *i.e.* the inclusion of explicit strange degrees of freedom like kaons, lambdas etc. Repeating the analysis of the isovector form factors of the nucleon in SU(3) HBChPT one finds an extra contribution

from kaon loops to $(r_2^v)^2$ (Eq.(2.2)) of the order of a few percent. This contribution vanishes if the strange quark mass becomes very heavy. For the case of the isovector nucleon form factors SU(3) HBChPT therefore reproduces our physical expectation that the kaon contributions are much less important than their pion counterparts due to the much larger mass. Here the pions clearly dominate the long-range physics and control the size of the nucleon. This is also seen in the spectral function [9].

However, this is not always the case. For example in the isoscalar form factors of the nucleon the leading chiral contribution to the radius is given by the kaon cloud, as the pionic contribution only begins via 3 pion intermediate states at the 2 loop level. The $\mathcal{O}(p^3)$ analysis suggests that roughly 30% of the isoscalar radius of the nucleon comes from structure related to the kaons in the nucleon! For details regarding these issues we refer to [1, 2, 9].

Another sector where explicit strange degrees of freedom figure prominently concerns–quasi by definition–the recent interest in the so-called "strangeness content of the nucleon", *e.g.* see ref.[10]. In the following we focus on the strangeness vector current of the nucleon defined as

$$\langle N| \, \bar{s} \, \gamma_\mu \, s \, |N\rangle = \langle N| \, \bar{q} \, \gamma_\mu \, (\lambda^0/3 - \lambda^8/\sqrt{3}) \, q \, |N\rangle = (1/3)J_\mu^0 - (1/\sqrt{3})J_\mu^8 \,, \quad (3.1)$$

with $q = (u, d, s)$ denoting the triplet of the light quark fields and $\lambda^0 = I$ (λ^a) the unit (the $a = 8$ Gell–Mann) SU(3) matrix. Assuming conservation of all vector currents, the corresponding singlet and octet vector current for a nucleon can then be written as

$$J_\mu^{0,8} = \bar{u}_N(p') \left[F_1^{(0,8)}(q^2)\gamma_\mu + F_2^{(0,8)}(q^2)\frac{i\sigma_{\mu\nu}q^\nu}{2M_N} \right] u_N(p) \,. \quad (3.2)$$

Here, $q_\mu = p'_\mu - p_\mu$ corresponds to the four–momentum transfer to the nucleon by the external singlet $(v_\mu^{(0)} = v_\mu \lambda^0)$ and the octet $(v_\mu^{(8)} = v_\mu \lambda^8)$ vector source v_μ, respectively. The strangeness Dirac and Pauli form factors are defined via

$$F_{1,2}^{(s)}(q^2) = \frac{1}{3} F_{1,2}^{(0)}(q^2) - \frac{1}{\sqrt{3}} F_{1,2}^{(8)}(q^2) \,, \quad (3.3)$$

subject to the normalization $F_1^{(s)}(0) = 0, F_2^{(s)}(0) = \kappa_B^{(s)}$, with $\kappa_B^{(s)}$ the (anomalous) strangeness moment. In the following we concentrate our analysis on the "magnetic" strangeness form factor $G_M^{(s)}(q^2)$, which in analogy to the (electro)magnetic Sachs form factor is defined as

$$G_M^{(s)}(q^2) = F_1^{(s)}(q^2) + F_2^{(s)}(q^2) \,. \quad (3.4)$$

In the case of a nucleon $G_M^{(s)}(0) \equiv \mu_N^{(s)}$ defines the so called "strange magnetic moment" of the nucleon whose sign/size is heavily contested in theoretical analyses. Furthermore, it is precisely this form factor at $q^2 = -0.1\text{GeV}^2$ which has been analyzed in the recent Bates measurement [12].

One expects that for low q^2 ChPT can give a prediction for this (as of 1998!) unknown quantity. For the case of $\mu_N^{(s)}$ this is only partially correct as one needs additional information about an unknown isosinglet counterterm [2, 11]. However, even if one cannot calculate the overall normalization of $G_M^{(s)}(q^2)$ at $q^2 = 0$, the evolution of this form factor with q^2 can be predicted in terms of well-known low energy quantities! To $\mathcal{O}(p^3)$ in SU(3) HBChPT one finds [2]

$$G_M^{(s)}(Q^2) \;\;=\;\; \mu_N^{(s)} + \frac{\pi M_N m_K}{(4\pi F_\pi)^2} \frac{2}{3}\left(5D^2 - 6DF + 9F^2\right) f(Q^2)\;, \quad (3.5)$$

with $Q^2 = -q^2, D \simeq 3/4, F \simeq 1/2, F_\pi \equiv (F_\pi + F_K)/2 \simeq 102\,\text{MeV}$ the average pseudoscalar decay constant and m_K being the kaon mass. The momentum dependence is given entirely in terms of the function

$$f(Q^2) = -\frac{1}{2} + \frac{4 + Q^2/m_K^2}{4\sqrt{Q^2/m_K^2}}\arctan\left(\frac{\sqrt{Q^2}}{2m_K}\right)\;. \quad (3.6)$$

For small and moderate Q^2 it rises almost linearly with increasing Q^2.

I emphasize that Eq.(3.5) only contains the leading order chiral contribution which stems exclusively from the kaon-cloud of the nucleon. It will be interesting to calculate the next-to-leading order (*i.e.* $\mathcal{O}(p^4)$) correction to this result in order to check possible contributions from vector mesons which are usually assumed to dominate this form factor [13]. However, already at $\mathcal{O}(p^3)$ one can implicitly include some of the higher order corrections if one analyzes the magnetic isoscalar form factor $G_M^{I=0}(Q^2)$ and the strange magnetic form factor $G_M^{(s)}(Q^2)$ simultaneously [2]. One obtains the *model-independent* connection

$$G_M^{(s)}(Q^2) = \mu_N^{(s)} + \mu_N^{I=0} - G_M^{I=0}(Q^2) + \mathcal{O}(p^4)\;, \quad (3.7)$$

where $\mu_N^{I=0}$ denotes the isoscalar magnetic moment of the nucleon. To $\mathcal{O}(p^3)$ one therefore predicts that the low Q^2 behavior of the strange magnetic form factor of the nucleon is *exactly* controlled by the well-known isoscalar form factor of the nucleon! For details I refer to [2].

Eqs.(3.5,3.7) can be considered as a lower, upper bound on the q^2 evolution of the strange magnetic form factor at low momentum transfer [2]. Both relations can be used to extrapolate from the experimentally determined values for $G_M^{(s)}(Q^2)$ at $Q^2 > 0$ to the sought after strange magnetic moment $\mu_N^{(s)}$ of the nucleon at $Q^2 = 0$. Clearly, with improving experimental accuracy on $G_M^{(s)}(Q^2)$ one also needs to calculate the $\mathcal{O}(p^4)$ corrections to both relations. Furthermore, comparing Eqs.(3.5,3.7) we are also looking forward to the mapping of the low Q^2 dependence of $G_M^{(s)}(Q^2)$ by the G0 collaboration at J-Lab [14].

4 The Isovector $N\Delta$ Transition

Finally, I want to give a brief update on the ongoing calculations [3, 15] regarding the isovector nucleon-delta transition multipoles and form factors. Recent interest is mainly triggered by three observations:

1. In a multipole analysis one finds that in the photoexcitation of $\Delta(1232)$ $[\gamma N \to \Delta]$ one can only have magnetic dipole (M1) or electric quadrupole (E2) transitions from the nucleon to the delta. Simple constituent quark models of the nucleon however generally assume all quarks to be in an s-wave state and therefore predict zero strength for the E2 transition. Several fits to pion photoproduction data in the delta region however show a non-zero ratio of E2/M1 strength of about -1% to -3% (*e.g.* [16]), indicating non-radial/many-body components in the ground-state wave function of the nucleon.

2. For electroproduction of $\Delta(1232)$ $[\gamma^* N \to \Delta]$ the transition multipoles M1,E2 do not only develop a dependence on the four-momentum transfer (squared) Q^2 but one can now have additional contributions from a Coulomb quadrupole transition C2. Our knowledge of the Q^2 dependence of these three multipoles for $0 < Q^2 < 1\text{GeV}^2$ mainly stems from experiments of the 1970s [17]. Recently, new measurements have started at Bonn which seem to validate the old analyses showing interesting differences in the Q^2 behavior among these multipoles for $Q^2 < 0.3\text{GeV}^2$ [18]. Furthermore, one would also like to compare the Q^2-falloff of the $N\Delta$ transition form factors with the well-known dipole behavior of the electric/magnetic Sachs form factors of the nucleon, *e.g.* see [17, 19].

3. Perturbative QCD predicts that for very large four-momentum transfer the ratio of E2/M1 for the case of delta electroproduction should tend to unity. At which finite Q^2 the crossover from a negative to a positive ratio should happen and whether this point is kinematically accessible at present/future electron scattering machines is an issue of current theoretical debate, *e.g.* [20].

We have started two collaborations [3, 15] to look into these topics from the viewpoint of ChPT. In particular, we are using the recently developed SSE formalism [7] in order to treat the delta resonance in a systematic fashion. What kind of results can one expect from these efforts?

1. There exist already 2 calculations regarding the ratio of E2/M1 at the real photon point utilizing ChPT [22]. Our present understanding is that one needs to take into account non-zero contributions from *three* different ingredients—namely pion loops, 1/M corrections and counterterms. While the loop contribution is relatively easy to calculate and agreed upon, the contributions from the 1/M corrections and counterterms have not been handled with the same accuracy so far. SSE offers a systematic formalism to address both aspects. At this point we can say that the actual number for E2/M1 in ChPT is quite sensitive to the treatment/size of several unknown counterterms. In order to settle this issue one needs a full calculation of pion-photoproduction in the delta resonance region [15] in order to fix these unknowns with the accuracy required for E2/M1. Only then one can expect a new systematic prediction for E2/M1 from

ChPT. We also note that in the past only the leading delta contribution to the s-wave multipole E_{0+} had been calculated explicitly in SSE [7]. The p-wave multipoles are known to receive large contributions from $\Delta(1232)$, but so far these effects have only been included via "resonance saturation" in higher order couplings [21]. Utilizing SSE [7], we are now analyzing explicit $\Delta(1232)$ components in the three p-wave multipoles. It will be interesting to see how far in energy the inclusion of explicit delta degrees of freedom can extend the applicability of ChPT to pion-photoproduction off nucleons into the delta resonance region. [15].

2. Surprisingly, the determination of the Q^2-evolution of the three $N\Delta$ transition form factors and of the corresponding three transition multipoles is a much simpler problem in ChPT, but has not been addressed so far. The important point to realize is that most of the unknown couplings/counterterms only concern the $Q^2 = 0$ values. Once one fixes the form factors/multipoles at the measured real photon values [16] one obtains their Q^2-dependence in terms of very few parameters which are under control. It is then straightforward to extract radii for the transition form factors and compare with the form factors of the nucleon. This project [3] is close to being finished once the problem of the scaling in the radii (discussed below) is fixed.

3. Concerning the third issue, ChPT can certainly not answer the problem of the onset of perturbative QCD in the E2/M1 ratio, probably even the zero-crossing point is at too high a momentum transfer for this approach. However, it should be possible to say whether E2/M1 first drops even more negative for low momentum transfer and whether there is a *turning point* after which the curve moves towards a positive value.

Finally, I want to address a problem that we encountered during the calculation of the isovector $N\Delta$ transition form factors. Assuming conservation of the vector current as well as invariance under P,C,T symmetry operations one concludes that in general there exist 3 independent structures for such a transition. To be more specific, let's assume that we are talking about the process $\Delta \to N\gamma^*$. The matrix-element is then typically written as [23][1]

$$
\begin{aligned}
i\mathcal{M}^{full}_{\Delta \to N\gamma} &= \frac{e}{2M_N}\bar{u}(p_N)\gamma_5\left[g_1(q^2)(\not{q}\epsilon_\mu - \not{\epsilon}q_\mu) + \frac{g_2(q^2)}{2M_N}(p_N \cdot \epsilon\, q_\mu\right.\\
&\left. -p_N \cdot q\,\epsilon_\mu) + \frac{g_3(q^2)}{2M_N}(q \cdot \epsilon\, q_\mu - q^2\epsilon_\mu)\right]u^\mu_\Delta(p_\Delta)\ . \quad (4.1)
\end{aligned}
$$

Here M_N is the nucleon mass, $p_{N,\Delta}$ denotes the momentum of the nucleon, delta and q, ϵ are the photon momentum and polarization vectors, respectively. The delta is described in the Rarita-Schwinger formalism, i.e. as an axial-vector spinor u^μ_Δ. Now one proceeds to calculate this matrix element in a

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[1] Minor differences to the form of Eq.(4.1) arise via field-redefinitions utilizing the equations of motion for the baryons. However, this does not change the thrust of the above argument.

non-relativistic microscopic approach, in our case SSE [7]. Calculating to third order in the expansion scheme the calculation can produce up to two inverse powers of the expansion scale, *i.e.* one is sensitive to structures up to $1/M_N^2$. In order to match the calculation with the most general matrix element Eq.(4.1) one also needs to expand it up to the same power in $1/M_N^2$. One finds

$$
\begin{aligned}
i\mathcal{M}_{\Delta->N\gamma}^{(3)} \;=\; & e\,\bar{u}_v(r_N)\left\{(S\cdot\epsilon)q_\mu\left[\frac{g_1(q^2)}{M_N}+\mathcal{O}(1/M_N^3)\right]\right. \\
& +(S\cdot q)\epsilon_\mu\left[-\frac{g_1(q^2)}{M_N}-\frac{\Delta}{2M_N^2}g_1(0)+\frac{\Delta}{4M_N^2}g_2(0)+\mathcal{O}(1/M_N^3)\right] \\
& +(S\cdot q)(v\cdot\epsilon)q_\mu\left[\frac{(g_1(0)-\frac{1}{2}g_2(0))}{2M_N^2}+\mathcal{O}(1/M_N^3)\right] \\
& \left.+(S\cdot q)(q\cdot\epsilon)q_\mu\left[0+\mathcal{O}(1/M_N^3)\right]\right\}u_{v,\Delta}^\mu(0)\;.
\end{aligned}
\tag{4.2}
$$

Here S_μ denotes the Pauli-Lubanski vector, v_μ corresponds to the velocity vector of the delta reference frame and $\Delta = M_\Delta - M_N$. As one can see from Eq.(4.2) the $1/M_N^2$-expansion demands that there are no explicit structures proportional to $\epsilon\cdot q$ to this order. Nevertheless the SSE calculations yield such terms! We therefore have to conclude that the often-used form for the isovector $N\Delta$ transition Eq.(4.1) is *not compatible* with (non-relativistic) microscopic calculations of this transition that rely on a systematic $1/M$ expansion. It is therefore mandatory to rescale $g_2(q^2), [g_3(q^2)]$ by $M_N/\Delta, [M_N^2/\Delta^2]$ in order to achieve a systematic matching between the microscopic calculations and the most general amplitude. Furthermore, without this rescaling of the form factors their transition radii would scale as $r_i^2 \sim M_N^n$; $i = 2, 3; n \geq 1$, *i.e.* one would see no $1/M$ suppression[2] in the radii compared to the $q^2 = 0$ point! Finally, we note that phenomenological analyses of data utilizing Eq.(4.1) are not affected by this problem, as long as all amplitudes are treated in a fully relativistic form. A detailed publication describing all these aspects is in preparation [3].

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